A Review: Counterparty Risk

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Abstract

This review gives formal definitions of the terms credit value adjustment (CVA) and debt value adjustment (DVA). To estimate these quantities requires modeling the probabilities of default and the loss given default, recognizing the dependence structure between all these inputs. In practice marginal distributions are used and a copula function assumed. While it has long been known that different copula functions can produce very different price estimates, keeping marginal distributions constant, there is little empirical evidence about the appropriate form of function to use for modeling default dependence. The use of collateral for risk mitigation and its effects on CVA are discussed. Regulators have argued that standardized contracts should be cleared through central clearing parties. However, there are arguments against central clearing.

JEL Codes: G01, G13, G20, G28

1 Introduction

Counterparty risk is prevalent in any form of bilateral contract, not just financial products. As non-financial contracts are rarely marked-to-market, the literature on counterparty risk is exclusively devoted to financial contracts. The financial crisis provided a clear demonstration of the importance of accounting for counterparty risk both at the level of individual firm and at the level of the economy. The crisis highlighted many different facets of counterparty risk: wrong way dependence, information asymmetry, default dependence and risk mitigation.

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During the credit crisis, two hedge funds run by Bears Stearns collapsed. In August 2007, BNP told investors that they would be unable to withdraw money from three funds due to difficulties in valuing the underlying assets because of a lack of liquidity. The end of 2007 saw monolines struggling to maintain their triple "A" credit rating. They had sold protection on municipal bonds and highly rated tranches backed by subprime mortgages. By the end of 2007, down grades of monolines had started to occur, forcing enhanced money market funds, pension funds and banks to write down the value of their holdings. All of these institutions faced wrong way exposure to monolines. They had purchased insurance from the monolines, yet as the probability of default of the monolines increased, the institutions were forced to write down the value of their holdings.

In September 2008, Lehman Brothers filed for Chapter 11 protection, the largest bankruptcy in U.S. history. At the time of default, approximately $400 billion of credit default swap credit protection had been written on Lehman Brothers. This caused problems for the protection sellers as they adjusted their portfolios to pay the protection buyers, adding further unrest to the turmoil in the markets. The company also had a large number of derivative contracts with thousands of counterparties. At the time of default, the company still had an investment grade rating and many counterparties were caught by surprise. The size of the default, the large number of counterparties and the surprise of the failure caused financial instability. For many counterparties there was information asymmetry about the state of Lehman and the full extent of its exposures.

In the same month, the Federal government had to provide AIG with $85 billion of aid. AIG Financial Products credit derivative portfolio had a net notional exposure of $505 billion - Barr (2008). The company had direct exposure to the subprime market. Moody's, S&P and Fitch all downgraded AIG. It also faced two other forms of risk. The counterparties could ask AIG to post collateral if the underlying assets declined in value or if AIG's credit rating declined. The swap contracts were marked-to-market (model) each day. Given the turbulent conditions, collateral requests started arriving in August 2007, the first of many requests. The company also had $1.4 billion of its own commercial paper maturing in September 2008 and was facing difficulties in rolling over its debt, given concerns about its credit worthiness. It was also facing challenges rolling over its repurchase agreements, due to concerns about the collateral it was posting. The travails of the monolines and AIG stem from a failure of understanding the nature of default dependence and its impact upon valuation and credit worthiness. There was also a failure to appreciate the wrong way dependence associated with collateral. The probability of default increased for the assets on which they had sold protection, forcing AIG to post more collateral while the value of its collateral decreased.

The credit crisis has enforced explicit recognition of the effects of counterparty risk via the credit value adjustment (CVA), defined as the price of counterparty risk. Apart from posting collateral, one of the ways to hedge this form of exposure and reduce capital requirements is via the use of credit default swaps, assuming swaps on the counterparty exist. However, this has been causing problems in the sovereign credit default market. Banks trading derivatives with sovereigns in order to reduce their exposure have been buying sovereign CDS, affecting both the level and volatility of spreads. Many market participants believe that for some markets, a third of the spread is due to CVA activity – see Murphy (2012). The general decline in credit worthiness of financial institutions has allowed the writing down of the value of debt liabilities, via a debt value adjustment (DVA). While the use of such an adjustment is a logical consequence of market valuation, it has been controversial.

The credit crisis has also precipitated unprecedented changes in financial regulation: in the U.S. the Dodd-Frank Wall Street Reform, the Consumer Protection Act 2009; in Europe the European Market Infrastructure Regulation and globally, Basel III. Part of this regulatory reform process has been to move over the counter trading of standardized derivatives, such as interest rate swaps and credit default swaps, to be cleared through central counterparties (CCPs). Such a move to central clearing will increase transparency and reduce connectivity. When Lehman Brothers defaulted it had thousands of counterparties. This number could have been reduced, if some of these contracts had been centrally cleared. While central clearing has been hailed as a major step in reducing systemic risk, it is not without its own set of problems.

Section 2 of the paper describes the basic valuation framework. We first consider the case of no counterparty risk, then one risky counterparty and finally both counterparties can default. Expressions for CVA and DVA are derived. The section ends by identifying the many different parameters that must be estimated. One of the major practical issues is the modeling CVA and DVA. The first topic we address in Section 3 is the modeling of the probability of default. Often in Calibration two assumptions are made: the intensity function used in calculating the probability of default is a piecewise line linear function; and second, the recovery rate is constant – O'Kane (2008, Chapter 7). Both assumptions are counterfactual. The second topic we discuss is the empirical evidence about the recovery rate and approaches to modeling. The final topic in this section is the modeling of default dependency. Section 4 deals with risk mitigation. First, the use of master agreements, and second the use of collateral and its impact on pricing. The last part of this section addresses some of the many different issues involving the clearing through central clearing parties. A summary is given in Section 5.

2 Identification of the Issues

Let (Ω, G, G_t, Q) be a probability space, where G_t denotes the flow of information. Let H_t denote the right continuous filtration generated by the default events and F_t the flow of information except defaults: $G_t = H_t \vee F_t$. Here we use the term default to denote a credit event that triggers a payment.

2.1 No Counterparty Risk

In a bilateral contract with maturity T, denote the two counterparties by X and Y. We assume that there is no probability that either of the two counterparties will default over the life of the contract. Let $C(t,T)$ denote the discounted cash flows associated with the contract. The value of the contract to party X is denoted by $V(t,T;X,Y)$, where

$$\begin{aligned} V(t,T;X,Y) &= E_t[C(t,T)] \\ &= -V(t,T;Y,X) \end{aligned}$$

where $V(t,T;Y,X)$ is the value of the contract to party Y and all expectations are under the pricing measure Q. The formulation is quite general; for example, the contract could be a foreign exchange forward contract or an interest rate swap. The contract could be written on a reference entity C that is subject to default, for example a credit default swap (CDS) written on a reference entity C, with default time τ_C. To value the contract it is necessary to model the cash flows associated with the contract. If default of the underlying entity occurs, it is necessary to model the recovery rate, $R(\tau_C)$. The recovery rate will depend on the conditions at the time of default. In other words, the recovery rate is a random variable depending on the state of the economy and the reference entity. Many investors during the credit crisis were reminded of the well known fact that recovery rates are negatively correlated with the frequency of defaults. For descriptions of pricing models applied to different instruments, see Bielecki and Rutkowski (2002) and Schönbucher (2003).

2.2 One Risky Counterparty

We now relax the assumption of no default risk associated with the counterparties. We consider two separate cases. First, there is no default risk associated with X and second, there is default risk associated with Y. The cases are not symmetric. We further assume that the contract is written on a reference entity subject to default risk. To be concrete, we refer to this contract as a credit default swap. There is a large literature incorporating the effects of counterparty risk on pricing: Duffie and Huang (1996), Jarrow and Turnbull (1997 (a) and (b)), Walker (2006), Brigo, Pallavicini and Papatheodorou (2011) and Crépey, Gerboud, Grbac and Ngor (2013). A survey of many of the different pricing models is given in Brigo, Morini and Pallavicini (2013).

2.2.1 Risky Counterparty Y

We now assume the possibility that party Y could default. There are two number of cases to consider. The first case is the reference entity defaults before party Y

$$A_1 = \{\tau_C \leq \tau_Y\}$$

The second case is when Y defaults before the reference entity C

$$A_2 = \{\tau_Y \leq \tau_C\}$$

Assuming the above cases are exhaustive, we have

$$1_{A_1} + 1_{A_2} = 1 \tag{1}$$

We make the technical assumption that simultaneous defaults do not occur almost surely under the natural probability measure and thus also under any equivalent measure.

If Y defaults during the life of the contract, to determine the mark to market value it is assumed that X enters into a new swap with same swap rate. It is also assumed that there is no counterparty risk with the new swap. The value of the swap is denoted by $V(\tau_Y, T)$. If $V(\tau_Y, T) \leq 0$, the swap has value to party Y and consequently party X makes a payment to Y. If $V(\tau_Y, T) > 0$, the swap has value to party X and consequently party X wants a payment of $V(\tau_Y, T)$. However, as Y is in default, X expects a payment of $R_Y(\tau_Y)V(\tau_Y, T)$ from Y, where $R_Y(\tau_Y)$ is the recovery rate.

Let $C(t, T)$ denote the swap discounted cash flows over the life of the swap in the ignoring of counterparty risk, $C^D(t,T)$ the discounted cash flows in the presence of counterparty risk and $A(t,s)$ the stochastic discount factor. Therefore,

$$\begin{aligned} C^D(t,T) &= C(t,T)1_{(A_1)} \\ &\quad +[C(t,\tau_Y) + A(t,\tau_Y)V(\tau_Y,T)1_{(V\ \leq\ 0)} + A(t,\tau_Y)R_Y V(\tau_Y,T)1_{(V\ >\ 0)}]1_{(A_2)} \end{aligned} \tag{2}$$

Now

$$C(t,T)1_{(A_2)} = [C(t,\tau_Y) + A(t,\tau_Y)V(\tau_Y,T)1_{(V\ \leq\ 0)} + A(t,\tau_Y)V(\tau_Y,T)1_{(V\ >\ 0)}]1_{(A_2)}$$

and substituting this expression into (2) and using (1), gives

$$C^D(t,T) = C(t,T) - (1-R_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\ >\ 0)}1_{(A_2)}$$

Therefore the value of the swap is

$$E_t[C^D(t,T)] = E_t[C(t,T)] - E_t[L_Y(\tau_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\ >\ 0)}1_{(A_2)}], \tag{3}$$

where $L_Y(\tau_Y) = 1 - R_Y(\tau_Y)$. The above result is derived Bielecki and Rutkowski (2002), Brigo and Capponi (2009) and Gregory (2012) among others. This is a very intuitive results; the second term on the right side is simply the present value of the loss given default if the counterparty defaults.

In practice there is a third case. The reference entity defaults first during the life of the contract. If X is the protection buyer, the payment to X is $(1 - R_C(\tau_C))$, assuming a notional value of one. However, before settlement, Y defaults and the resulting payment is $(1 - R_C(\tau_C))R_Y(\tau_Y)$, where $\tau_C < \tau_Y \leq \tau_C + \Delta$, Δ denoting the settlement period.

$$A_3 = \{(\tau_C < \tau_Y) \cap (\tau_C < \tau_Y \leq \tau_C + \Delta) \cap (\tau_C \leq T)\}$$

The loss given default is $(1 - R_C(\tau_C))(1 - R_Y(\tau_Y))$. If X is the protection seller, the loss given default is $S(\tau_C - T_j)(1 - R_Y(\tau_Y))$, where T_j is the last payment before the reference entity defaults. Define

$$L_{CY}(\tau_C,\tau_Y) = \{ \begin{array}{ll} (1 - R_C(\tau_C))(1 - R_Y(\tau_Y)) & ; \ \text{if } X \text{ is a protection buyer} \\ S(\tau_C - T_j)(1 - R_Y(\tau_Y)) & ; \ \text{if } X \text{ is a protection seller} \end{array} \tag{4}$$

Therefore,

$$1_{A_1} + 1_{A_2} + 1_{A_3} = 1$$

In this case we have

$$C^D(t,T) = C(t,T) - L_Y(\tau_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\ >\ 0)}1_{(A_2)} - L_{CY}(\tau_C,\tau_Y)A(t,\tau_Y)1_{(A_3)}$$

Therefore the value of the swap is

$$\begin{aligned} E_t[C^D(t,T)] &= E_t[C(t,T)] - E_t[L_Y(\tau_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\ >\ 0)}1_{(A_2)}] \\ &\quad -E_t[L_{CY}(\tau_C,\tau_Y)A(t,\tau_Y)1_{(A_3)}] \end{aligned} \tag{5}$$

Compared with (3) we have an additional term. The probability of event A_3 will typically be small; the reference entity has to default during the life of the contract and counter party Y during the settlement period. While this probability may be small, it magnitude could increase during credit crises when there is contagion - see Turnbull (2005).

Credit Value Adjustment CVA

The CVA is a measure of how the value of a contract is affected by counterparty risk and is defined by

$$E_t[C^D(t,T)] \equiv E_t[C(t,T)] - CVA(t,T)$$

where $CVA(t,T)$ denotes the *credit value adjustment* due to counterparty risk. From (5), we have

$$\begin{aligned} CVA(t,T) &= E_t[L_Y(\tau_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\ >\ 0)}1_{(A_2)}] \\ &\quad +E_t[L_{CY}(\tau_C,\tau_Y)A(t,\tau_Y)1_{(A_3)}] \end{aligned} \tag{6}$$

2.2.2 Risky Counterparty X

We now assume the possibility that party X could default. There are two number of cases to consider. The first case is the reference entity defaults before party X

$$A_1 = \{\tau_C \leq \tau_X\}$$

The second case is when X defaults before the reference entity C

$$A_2 = \{\tau_X \leq \tau_C\}$$

If $V(\tau_X, T) > 0$ the swap has value to X and Y makes a full payment. If $V(\tau_X, T) < 0$ the swap has value to Y and X makes a payment $-R_X(\tau_X)V(\tau_X, T)1_{(V<0)}$ to Y. The above cases are exhaustive,

$$1_{A_1} + 1_{A_2} = 1$$

Therefore, by repeating the arguments given in the last section, we have

$$C^D(t,T) = C(t,T) - (1 - R_X)A(t,\tau_X)V(\tau_X,T)1_{(V\ <\ 0)}1_{(A_2)}$$

and the value of the swap is

$$E_t[C^D(t,T)] = E_t[C(t,T)] - E_t[L_X(\tau_X)A(t,\tau_X)V(\tau_x,T)1_{(V\ <\ 0)}1_{(A_2)}] \tag{7}$$

Repeating the argument from the last section, there is a third case to consider. The reference entity defaults first during the life of the contract. If X is the protection buyer, the payment to X is $(1 - R_C(\tau_C))$, assuming a notional value of one and Y expects $S(\tau_C - T_j)$. However, before settlement, if X defaults and the resulting payment is $S(\tau_C - T_j)R_X(\tau_X)$, where $\tau_C < \tau_X \leq \tau_C + \Delta$, Δ denoting the settlement period and T_j is the last payment before the reference entity defaults

$$A_3 = \{(\tau_C < \tau_Y) \cap (\tau_C < \tau_X \leq \tau_C + \Delta) \cap (\tau_C \leq T)\}$$

The loss given default is $S(\tau_C - T_j)(1 - R_X(\tau_X))$. If X is the protection seller, the loss given default is $(1 - R_C(\tau_C))(1 - R_X(\tau_X))$. Define

$$L_{CX}(\tau_C, \tau_X) = \{ \begin{array}{ll} S(\tau_C - T_j)(1 - R_X(\tau_X)) & ;\ \text{if } X \text{ is a protection buyer} \\ (1 - R_C(\tau_C))(1 - R_X(\tau_X)) & ;\ \text{if } X \text{ is a protection seller} \end{array} \tag{8}$$

Therefore,

$$1_{A_1} + 1_{A_2} + 1_{A_3} = 1$$

In this case we have

$$\begin{aligned} C^D(t,T) &= C(t,T) - L_X(\tau_X)A(t,\tau_X)V(\tau_X,T)1_{(V\ <\ 0)}1_{(A_2)} \\ &\quad - L_{CX}(\tau_C,\tau_X)A(t,\tau_X)1_{(A_3)} \end{aligned}$$

Therefore the value of the swap is

$$\begin{aligned} E_t[C^D(t,T)] &= E_t[C(t,T)] - E_t[L_X(\tau_X)A(t,\tau_X)V(\tau_X,T)1_{(V<\ 0)}1_{(A_2)}] \\ &\quad -E_t[L_{CX}(\tau_C,\tau_X)A(t,\tau_X)1_{(A_3)}] \end{aligned} \tag{9}$$

Compared with (7) we have an additional term.

2.2.3 Debt Value Adjustment (DVA)

From the position of X, if it defaults before Y and if $V(\tau_X, T)$ is negative, then X owes Y this amount. However, Y will only receive $R_X(\tau_X)V(\tau_X, T)$, implying that X saves an amount $[1 - R_X(\tau_X)]V(\tau_X, T)$ by defaulting. The risk of defaulting by X should be considered in fair value measurement just as counterparty risk is considered. The debt value adjustment is defined by

$$DVA(t,T) \equiv E_t[L_X(\tau_X)A(t,\tau_X)V(\tau_X,T)1_{(V\ <\ 0)}1_{(A_2)}] \tag{10}$$

The recording of DVA is controversial, as it is hard to monetize and counter intuitive to some - see Brigo and Morini (2011) and Gregory and German (2013). It increases in value as the credit quality of X deteriorates[1].

Since November 2007, Financial Accounting Standards 157 and 159 requires all financial derivative transactions to be reported at fair value and under International Financial Reporting Standards (IFRS) 13, fair value accounting had to be reported effective January 1, 2013. Both CVA and DVA are major contributors to trading revenue volatility - see Lopez and Rodriguez (2013). In the first quarter of 2013 Citigroup[2] reported \$310 million of CVA/DVA and Credit Agricole[3] reported average one day CVA of minus 296 million euros and DVA of 250 million euros. For year 2012, JP Morgan Chase reported a gain of \$1.4 billion from DVA and a loss of \$769 million from CVA.

[1] See Ernst & Young (2012) for survey results from 19 financial institutions.
[2] Citigroup Press release April 16, 2013.
[3] First Quarter Report, Credit Agricole S.A. (2013).

2.3 Two Risky Counterparties

We now assume the possibility that either party X or Y could default. There are a number of cases to consider. First, the reference entity C before parties X and Y.

$$A_1 = \{\tau_C \leq \tau_X) \cap (\tau_C \leq \tau_Y)\}$$

The second case is when Y defaults before the reference entity and before X

$$A_2 = \{(\tau_Y \leq \tau_C) \cap (\tau_Y \leq \tau_X)\}$$

The third case is when X defaults before the reference entity and before Y

$$A_3 = \{(\tau_X \leq \tau_C) \cap (\tau_X \leq \tau_Y)\}$$

If X defaults during the life of the contract and if $V(\tau_X, T) > 0$, implying that the contract has positive value to X, then X receives the full payment from Y. If $V(\tau_X, T) < 0$, then the contract has positive value to Y. However, Y will only receive $-R_X(\tau_X)V(\tau_X, T)1_{(V<0)}$. For the moment we ignore the case when the reference entity defaults during the life of the contract and before settlement, Y defaults. For the moment, we assume the above cases are are exhaustive. Therefore,

$$1_{A_1} + 1_{A_2} + 1_{A_3} = 1$$

Therefore, repeating the type of argument in the last section, we have

$$\begin{aligned} C^D(t,T) &= C(t,T) - (1 - R_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\,>\,0)}1_{(A_2)} \\ &\quad -(1 - R_X(\tau_X))A(t,\tau_X)V(\tau_X,T)1_{(V\,<\,0)}1_{(A_3)} \end{aligned}$$

and the value of the swap is

$$\begin{aligned} E_t[C^D(t,T)] &= E_t[C(t,T)] - E_t[L_Y(\tau_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\,>\,0)}1_{(A_2)}] \\ &\quad -E_t[L_X(\tau_X)A(t,\tau_X)V(\tau_X,T)1_{(V\,<\,0)}1_{(A_3)}] \end{aligned}$$

The credit value adjustment is

$$\begin{aligned} CVA(t,T) &= E_t[L_Y(\tau_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\,>\,0)}1_{(A_2)}] \\ &\quad +E_t[L_X(\tau_X)A(t,\tau_X)V(\tau_X,T)1_{(V\,<\,0)}1_{(A_3)}] \end{aligned}$$

Note that the above analysis is from the view of X. From the view point of Y, the credit value adjustment is $-CVA(t,T)$.

In practice there is a two other cases to consider: the reference entity defaults and then Y (X) defaults before settlement. The first case is described by

$$A_4 = \{(\tau_C < T < \tau_Y < \tau_X) \cap (\tau_C < \tau_Y \leq \tau_C + \Delta)\}$$

with loss given default defined by (4) and the second case

$$A_5 = \{(\tau_C < T < \tau_X < \tau_Y) \cap (\tau_C < \tau_X \leq \tau_C + \Delta)\}$$

with loss given default defined by (8). We now have

$$1_{A_1} + 1_{A_2} + 1_{A_3} + 1_{A_4} + 1_{A_5} = 1$$

and the value of the swap is

$$\begin{aligned} E_t[C^D(t,T)] &= E_t[C(t,T)] - E_t[L_Y(\tau_Y)A(t,\tau_Y)V(\tau_Y,T)1_{(V\,>\,0)}1_{(A_2)}] \\ &\quad - E_t[L_X(\tau_X)A(t,\tau_X)V(\tau_X,T)1_{(V\,<\,0)}1_{(A_3)}] \\ &\quad - E_t[L_{CY}(\tau_C,\tau_Y)A(t,\tau_Y)1_{(A_4)}] \\ &\quad - E_t[L_{CX}(\tau_C,\tau_X)A(t,\tau_X)1_{(A_5)}] \end{aligned} \tag{11}$$

2.4 Issues

To value a CDS in the absence of counterparty risk a model is required to determine the probability of surviving over a given period and to model the recovery rate of the reference entity if default occurs. In general the probability of default and the recovery rate are state dependent. For example, if the economy goes into recession, then it is often observed that the probability of default increases, while the recovery rate decreases. For trading purposes and risk management, the model must be consistent with market prices. In the presence of counterparty risk, the model becomes more complicated, as now it is required to model the survival probabilities of the two counterparties and their respective recovery rates. Furthermore, it is necessary to recognize the default dependence among the parties. The last issue is how are the parameters of the model estimated?

3 Modeling CVA and DVA

We start by considering the case of only one counterparty being at risk of default, say party Y. Examining expression (6) we must estimate the loss given default at the time τ_Y and the probability of Y defaulting before the reference entity over the life of the contract.

3.1 Modeling the Probability of Default

Two approaches have been used to model the probability of a credit event by an obligor. The first approach is the structural model introduced by Merton (1974) and Black and Cox (1976). The Merton model takes the market value of a firm as exogenous. A firm is assumed to have simple capital structure, with only one type of debt: zero coupon that matures at on a fixed date. Given the face value of the debt, then the probability of default under the pricing measure can be determined. Black and Cox relax the assumption that default can only occur at one point, the maturity of the debt, by assuming a barrier model. The first time the value of the firm drops below a critical value, default is assumed to occur. The barrier is exogenous. The basic model has been extended in many directions: stochastic interest rates Longstaff and Schwartz (1995); strategic default, Anderson and Sundaresan (1996) and Mella-Baral and Perraudin (1997); endogenous barrier, Leland (1994) and Leland and Toft (1996); jump-diffusion models, Kou and Wang (2003) and Cremers, Driessen and Maenhout (2008); and jump to default models, Linetsky (2006) and Carr and Madan (2010). The ability of this class of models to explain credit spreads is questioned in Eom, Helwege and Huang (2004), Huang and Zhou (2008) and Huang and Huang (2012).

Reduced models for pricing credit risky assets were introduced by Jarrow and Turnbull (1992, 1995). The intensity function must be strictly positive, a Feller process of the form

$$d\lambda_j(t) = \kappa_j(\mu_j - \lambda_j(t))dt + \sigma_j\sqrt{\lambda_j(t)}dZ_j(t) \tag{12}$$

is a usually assumed - see Duffie and Singleton (1999), where for obligor j, $Z_j(t)$ is a Brownian motion, κ_j, μ_j, σ_j and $\lambda_j(0)$ are constants that must be specified. The probability of surviving over a given period t, is given by

$$Q[\tau_j > t] = E[\exp(-\int_0^t \lambda_j(u)du)] \tag{13}$$

A closed form solution for the above expression is given in Duffie and Singleton (1999). One extension is to add jumps to the basic specification (12) - see Duffie, Pan and Singleton

(2000). This adds two more parameters (jump frequency and size of jump) to the number of parameters to estimate. Latent factor models such as (12) have been used for pricing corporate bonds - see Duffie and Singleton (1999), Duffee (1999), Driessen (2005) and Feldhuttee and Lando (2005). For credit default swaps see Houweling and Vorst (2005), Longstaff, Mittal and Neis (2005) and Chen, Cheng, Fabozzi and Liu (2008).

Lando (1994,98) assumed the default intensity function is a Cox process depending on covariates. In an arbitrage free framework, Bakshi, Madan and Zhang (2006) using bond data and Doshi, Ericsson, Jacobs and Turnbull (2013) using credit default swap data, show that macro economic and firm specific covariates are important determinants of credit spreads.

3.2 Recovery Rates

Empirical evidence using actual recovery rates, shows that recovery rates tend to declines as the probability of default increases, implying that recovery rates are state dependent - see Altman, Brady, Rest and Sironi (2005), Schuermann (2004) and Acharya, Bharath and Srinivasan (2007). Both Acharya et al (2007) and Chava, Stefanescu and Turnbull (2011) find that contract characteristics, firm and macro economic variables and sector affect recovery rates.

Under the pricing measure there is less empirical evidence. Pan and Singleton (2008) estimate constant recovery rates using sovereign credit default swap data for three countries. The assumption of a constant recovery rate is inadequate for pricing senior tranches, as it can imply they are free from default risk. Zhang (2010) using corporate bond data, shows that there is empirical support in specifying the recovery rate in the form

$$R_j = w_{0j} + w_{1j} \exp(-\lambda_j)$$

where w_{0j} and w_{1j} are constants satisfies $0 \leq w_{0j} + w_{1j} < 1$. Amraoui, Cousot, Hitier and Laurent (2009) make a similar assumption. Jaskowski and McAleer (2012) estimate a similar model using CDS data for three US firms. It should be observed that the above formulation assumes that the intensity function is a sufficient statistic for the recovery rate. The empirical evidence in Chava et al (2011) shows that this assumption is problematic.

There is strong empirical evidence that the recovery rate (and hence loss given default) is state dependent. This state dependency should be considered when estimating the effects of counterparty risk. In a Gaussian copula latent factor framework, Andersen and Sidenius (2004) assumed that for each obligor the event of default depends on a small number of factors common to all obligors plus an obligor specific factor. A similar assumption is made to describe the recovery rate in the event of default. This allows them to describe the negative correlation between recovery rates and default frequencies. Li (2013) surveys recent work.

3.3 Default Dependence

Factors models have been widely used to model default dependence. Drawing on Vasicek (1987), the value of the firm is represented by a linear function of a latent factor common to all obligors and an independent idiosyncratic component. The two factors are assumed to be described by independent, zero mean, unit variance normally distributed random variables. Lucas (2001) replaces the assumption of a single common factor with a vector of common dependent factors, described by a multivariate normal distribution with zero mean and a covariance matrix. The assumption of multivariate normal distribution can be replaced with a multivariate t-distribution. This will lead to fatter tails compared to the multivariate normal distribution.

If, for each obligor, the default intensity is described by a Cox process that depends on macro and obligor specific factors, then default intensities will be correlated generating default dependence. Schönbucher (2003) argues such models are not capable of generating sufficient dependence. However, Yu (2007) argues that with appropriate covariates in the specification of the Cox process, sufficient levels of default dependence can be generated. Duffie and Garleanu (2001) specify the intensity function to be a linear function of independent latent factors plus an idiosyncratic term. All factors are assumed to be described by Feller processes. Practical implementation is not addressed.

Instead of directly specifying a multivariate distribution of covariates (observable or latent), an alternative approach is to specify a copula function. A copula function maps the marginal distributions into the multivariate distribution. McNeil, Frey and Embrechts (2005) provide a good introduction to the different types of copula functions and their properties used in finance - see also Nelsen (1999). For hedging and counterparty risk, marginal distributions are calibrated to match extant prices. The attraction of the copula approach is that it isolates the dependence structure from the marginal distributions.

The Gaussian copula function, first introduced by Li (2000) for modeling default dependence, has become an industry standard. Apart from the marginal distributions, it is necessary to specify a covariance matrix. While equity factor models are used to estimate the covariance matrix, there is little empirical evidence to justify such a choice. It is implicitly assumed that the factors that drive equity returns are identical to the factors that drive defaults. One of the draw backs of the Gaussian copula is that it does not exhibit upper or lower tail dependence for typical values of correlation.

To introduce tail dependence other forms of copula must be used. Burtschell, Gregory and Laurent (2005, a) describe extensions of the Gaussian copula model that allows stochastic correlation. There are an infinite number of copula functions that can be used and the choice of copula can have a large impact on the pricing of correlated products. Mashal, Naldi and Zeevi (2003) compare the pricing of different types of credit derivatives using the normal copula and t copula. Using maximum likelihood to estimate the parameters of the copula functions, they show that the differences in prices for first and second to default basket derivatives can be substantial.

Parameters can be estimated using either calibration or historical data. If calibration is employed, then parameters are chosen to perfectly match current prices. The parameter estimates reflect the current market conditions. Traders prefer any model and its associated greeks to be consistent with extant prices and information. If historical data are used, then the parameters are estimated using data over a particular period. The parameter estimates will reflect the variation in prices over that period.

The method of calibration can affect the relative performance of models. Burtschell, Gregory and Laurent (2005, b) provide a comparative analysis of synthetic CDO pricing models. Parameters for different copula models are calibrated so that there is agreement on the pricing of the equity tranche. The mezzanine and senior tranches were then priced. The Gaussian, Student t, Clayton copula models produced similar prices. This contradicts the Mashal, Naldi and Zeevi (2003) finding, because of the different methods of calibration. The Marshall-Olkin copula produced quite different prices. For matching observed market prices, again models were calibrated to match the equity tranche. The Gaussian, Student t, and Clayton copula models, while producing similar prices, did not match the market prices for the other tranches. The double $t-$ copula, introduced by Hull and White (2004), did a better job of matching prices. However Kalemanova, Schmid and Werner (2005) show that using an inverse normal does have a slightly better fit than the double t- copula model and it is five times faster. Wang, Rachev and Fabozzi (2009) show that for the double t copula model, instead of restricting the degrees of freedom to be integer, as in Hull and White, a superior fit is obtained by allowing the degrees of freedom to be a positive real number. Furthermore, the optimal degrees of freedom changes period by period.

For counterparty risk the challenge is to attach values to the parameters of the models estimating CVA. Practitioners usually calibrate a model to fit observed prices. However, both the choice of prices used for calibration and the method of calibration, affect the parameter values.

The Gaussian copula does not display lower or upper tail dependence for typical values of correlation, so it is not surprising that t copula can do a better job of matching market prices. The empirical evidence that the degrees of freedom varies over time, implies that extant models are incorrectly specified, as they treat the parameter as a constant. There is a lack of empirical work in this area about robust alternatives.

When a default occurs, this may affect remaining obligors. Jarrow and Yu (2001) study a case of primary and secondary firms. The credit worthiness of the secondary firm depends on a primary firm, but not vice versa. This allows them to mathematically analyze the pricing of claims. In general, if an obligor in an underlying structure such as a CDO defaults, this will affect the dependent structure – see Schönbucher (2003), the valuation of the structure and the effects of counterparty risk. Modeling this form of contingency requires strong assumptions and estimation of the parameters is problematic – see Crépey, JeanBlanc and Wu (2013).

3.4 Non-Linear Products

Most non-linear products, such as collateralized debt obligations, do not have closed form solutions, implying that Monte Carlo simulation must be used to estimate the value of the underlying instrument. The requirement to value the instrument via simulation greatly complications the calculation of the effects of counterparty risk. When one of the counterparties defaults, then conditional on this default, Monte Carlo simulation is used to estimate the value of the underlying instrument. But default by the counterparties can occur any time, implying that a Monte Carlo simulation within a simulation must be performed. Mashal and Naldi (2005) avoid this computational complication by deriving upper and lower bounds. The bounds are quite tight for low levels of risk. Turnbull (2005), using the MN methodology, shows the effects on $P\&L$ due to counterparty risk can be substantial, especially if the counterparties have high yield ratings. Hasse, Ilg, and Werner (2010) use a different criteria to derive upper and lower bounds. The issues of calibration and sensitivity of the results to underlying assumptions remain unexplored.

The approach of deriving upper and lower bounds can circumvent major computational challenges. However, to be useful, the bounds must be tight.

4 Risk Mitigation

At the obligor level, a loan officer has a number of tools with which to reduce the exposure to a counterparty. The probability of a default can be reduced by obtaining a guarantee from a third party. In the event of default the creditor status is sufficiently high, so as to improve the recovery rate. The use of ISDA Master agreements (see Harding (2002)) facilitates the netting of deal exposures. Netting is only effective if there are sufficient deals with negative mark-to-market to off positive exposure. In general there is always residual exposure. This risk can be mitigated via the use of collateral.

4.1 Collateral

To help mitigate the effects of counterparty risk, the parties negotiate on the conditions governing the size of collateral payments, frequency of payments, type of collateral. All of the conditions are described in the Collateral Support Annex (CSA), appended to the ISDA Master Agreement. The CSA will specify (a) the threshold amount, which defines the level of exposure above which collateral is posted; (b) the independent amount defining the collateral that must be posted independent of exposure; (c) the minimum transfer amount defining the minimum amount of collateral that can be requested. All transfer amount are rounded off. (d) The frequency of transfers will depend on the volatility of the underlying asset, the sophistication of the counterparty and the type of collateral. (e) The determination of eligible collateral, either cash or securities. If securities, then it is desirable that their value not be correlated with the value of the transaction. The specification of the CSA is subject to negotiation between the parties. The threshold and minimum transfer amounts may be linked to indicators of credit worthiness, such as credit ratings - see Bielecki, Cialenco and Iyigunler (2013) and Zhou (2013).

While collateral can in theory reduce credit exposure, it does generate new forms of risk: market, operational and liquidity. The New York CSA allows the receiver of collateral to use the collateral, though the pledger of the collateral retains ownership of the assets used as collateral. The receiver of the collateral can rehypothecate any posted collateral it holds. Rehypothecation lowers traders's funding liquidity needs and improves market liquidity. However, it may induce market wide counterparty risk, as explained in Monnet (2011). At the time of the failure of Lehman Brothers International (Europe), it was holding significant customer assets that could not be immediately returned. This caused liquidity issues. Pykhtin and Sokol (2013) examine the impact of default by a large (financial) counterparty, such as Lehman Brothers, and show that it can have a major impact on other financial institutions and credit markets. The Dodd-Frank Act (2010) has placed restrictions on the use of rehypothecation.

Following Gregory (2012, *ch.*6), the amount of collateral required at time t, ignoring for the moment minimum transfer amounts is given by

$$L_t = \max(V_t - H_{A,t}, 0) - \max(-V_t - H_{B,t}, 0) - C_{t-1}$$

where V_t is the mark-to-market, $H_{A,t}$ is the threshold amount, as determined by the A, $H_{B,t}$ is the threshold amount, as determined by the B and C_{t-1} the current value of the amount of collateral determined the last reset date. If L_t is above the minimum transfer amount, then new collateral is posted. If L_t is positive (negative) then B (A) is required additional collateral. Once a collateral request has been made, the counterparty must agree with the request. After any disputes are settled, there is a settlement period, its duration depends on the type of collateral. If collateral is not received on a timely basis, there may be a grace period before the counterparty is deemed to be in default. The possible delays in the delivery of collateral must be considered when estimating the effects of collateral in mitigating counterparty risk.

Cherbini (2005), drawing on the work of Sorensen and Bollier (1994), uses a replicating argument to consider the effects of collateral. The issues of minimum transfer amounts and rehypothecation are ignored. Assefa et al (2011) does not consider minimum transfer amounts. Alavian (2008) assumes that collateral is risk free. Brigo, Capponi, Pallavicini and Papatheodorou (2011), while considering minimum transfer amounts and rehypothecation, assume no delay in responding to collateral calls. While these studies provide a framework, details about actual implication - calibration of all the parameters, the assumptions behind modeling default dependence - are scare. There is no information about the relative sensitivity arising from different assumptions.

4.2 Central Clearing Parties (CCPs)

Since the failure of Lehman Brothers and the resulting financial crisis, politicians and regulators have argued that OTC derivative contracts are a threat to financial stability and such contracts should be cleared through a central clearing party. The European central Bank and the Dodd-Frank Act published formal proposal that all standardized OTC derivatives should be cleared through CCPs. Basel III provides an incentive to use central clearing: it has a lower capital charge for CCP cleared trades than for bilateral trades.

When two parties trade an instrument that is cleared through a CCP, the CCP imposes itself between the two parties and assumes all contractual rights and responsibilities, using a legal process is known as novation. After settlement, each party does not need to be concerned about the credit worthiness of the other party; it need only be concerned about the credit worthiness of the CCP. This is assumed to negligible, given that CCP are regulated. Most CCP require both members and customers to post collateral, usually cash or highly liquid instruments, and adjust collateral on a daily basis based on mark-to-market. The appeal of central clearing is simple: by mandating derivative contracts are cleared through a CCP, the effects on the economy of a dealer failing will be reduced. Such logic is intuitively appealing. If a dealer fails, only the exchange will be directly affected, connectivity has been reduced and given investor and dealer margins, the loss should be manageable. A CCP does not make counterparty risk disappear: it centralizes it in one place. There is always the possibility the CCP might fail.

The introduction of a clearing house introduces moral hazard and adverse selection issues and liquidity concerns - Bernanke (1990). If one member of a clearing arrangement defaults, the remaining members of the exchange are exposed to the resulting liability. This generates a moral hazard, as a dealer can increase the risk of its balance sheet, thereby increasing the likelihood that they will default on the clearing obligations. The costs arising from a dealer default are borne by the remaining members. The precise amount depends on the design of the exchange. It is costly to control moral hazard. The CCP typically uses collateral requirements to control moral hazard. The heterogeneity of dealers increases the costs of using margins to control risk. Moral hazard is also present in OTC contracts. Collateral arrangements for OTC bilateral contracts offer greater flexibility (compared to CCP) and can be designed to reduce cash flow volatility with collateral calls on a less frequent basis. Many small firms do not have the in house resources to deal with daily margin calls that would occur if cleared through a CCP. Issues arising from moral hazard are priced into the contract, reducing their severity. Pirrong (2010) provides a careful summary of the many issues associated with central clearing facilities.

Dealers who are trading instruments, have the incentives to develop pricing and risk models and typically will be better informed that a CCP about the balance sheet risks of other dealers. These informational asymmetries between the member dealers and the CCP, expose the CCP to adverse selection. A deal with superior information indicating that the CCP has underestimated balance sheet risk and hence charged an insufficient margin will have an incentive to increase trading. As the number of different types of contracts increases, the CCP will face greater adverse selection, due to informational asymmetries.

Netting in bilateral contracts lowers the amount of required collateral. However, it does not allow for the netting of offsetting positions across "rings" of three or more trades – Gregory (2012). Such offsetting does occur when trades are cleared through a CCP. It can also occur through the use of compression trades – O'Kane (2013). However the multilateral netting that occurs through a CCP does not in itself justify the adoption of clearing, as shown by Duffie and Zhu (2010) and Pirrong (2010). There is a trade-off between the reduction in collateral via bilateral netting and multilateral netting via a CCP. If a CCP exists for one type of instrument, Duffie and Zhu (2010) and Cont and Kokholm (2012) show that adding another CCP for a different instrument can decrease overall exposure. The argument for clearing becomes more persuasive the greater the number of different types of instruments that can be cleared through a single CCP. However, scale will not be achieved if there is political pressure for trades to cleared in the country of origin – European Central Bank (2009).

The increase reliance on collateral as mandated under the Dodd-Frank Act, requires market participants to tie up increased amounts of low yield liquid assets, which has a real cost in terms of return foregone by traders. The greater the fragmentation of CCP across products and jurisdictions, the greater will be the required amount of collateral. This increased demand for collateral encourages the shadow banking system to create assets that are acceptable as collateral. These assets may embed tail risk, as explained by Pirrong (2012). The use of margins requirements makes the system pro-cyclical. CCPs tend to increase initial margins after large price moves associated with increased volatility. Capital constrained traders, who experienced a loss, typically respond to such margin increases by reducing positions. This tends to reinforce the price movements that caused the initial loss.

Bernanke (1990) provides an analysis of the 1987 stock market crash and argues that the Federal Reserve played a major role in supporting CCPs. Pirrong (2012) argues that under the Dodd-Frank Act, the clearing and collateral mandates during times of crisis have the potential to drain the system of liquidity, increasing systemic risk.

5 Summary

While there is a rich literature examining the mathematical modeling of counterparty risk, the underlying assumptions of these models are made for mathematical ease, independent of empirical evidence. For effective management and regulation, we need to be cognizant of the empirical evidence justifying underlying assumptions. Given the work of Wang, Rachev and Fabozzi (2009), we know that parameters of a copula function can change over time. How do we incorporate this finding into our models used to estimate CVA? How do we determine the appropriate form of copula function to use when modeling default dependence ? What is the sensitivity of CVA estimates to the different modeling assumptions? Arora, Gandhi and Longstaff (2012) make the pithy observation that there is relatively little empirical research about how counterparty risk affects prices. We can add that there is little empirical evidence justifying the estimates of CVA.

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